

## Mathematics Placement Packet

### Colorado College Department of Mathematics and Computer Science

Colorado College has two all college requirements (QR and SI) which can be satisfied in full, or part, by taking mathematics, statistics and computer science courses. There are also numerous majors and minors at CC which require some calculus or statistics (see the bottom of this document for a list of such majors). This worksheet and packet are designed to help you determine what might be a good first mathematics, statistics or computer science course for you.

Work your way through the decision tree on the next page. Consider the questions carefully and ask for advice or assistance if you are unsure. Use the following questions to help you as well. Note that the decision tree refers to three attached documents titled, *Sample Pre-Calc*, *Sample Calc I* and *Sample Calc II*. Please look at and read these documents carefully, when appropriate.

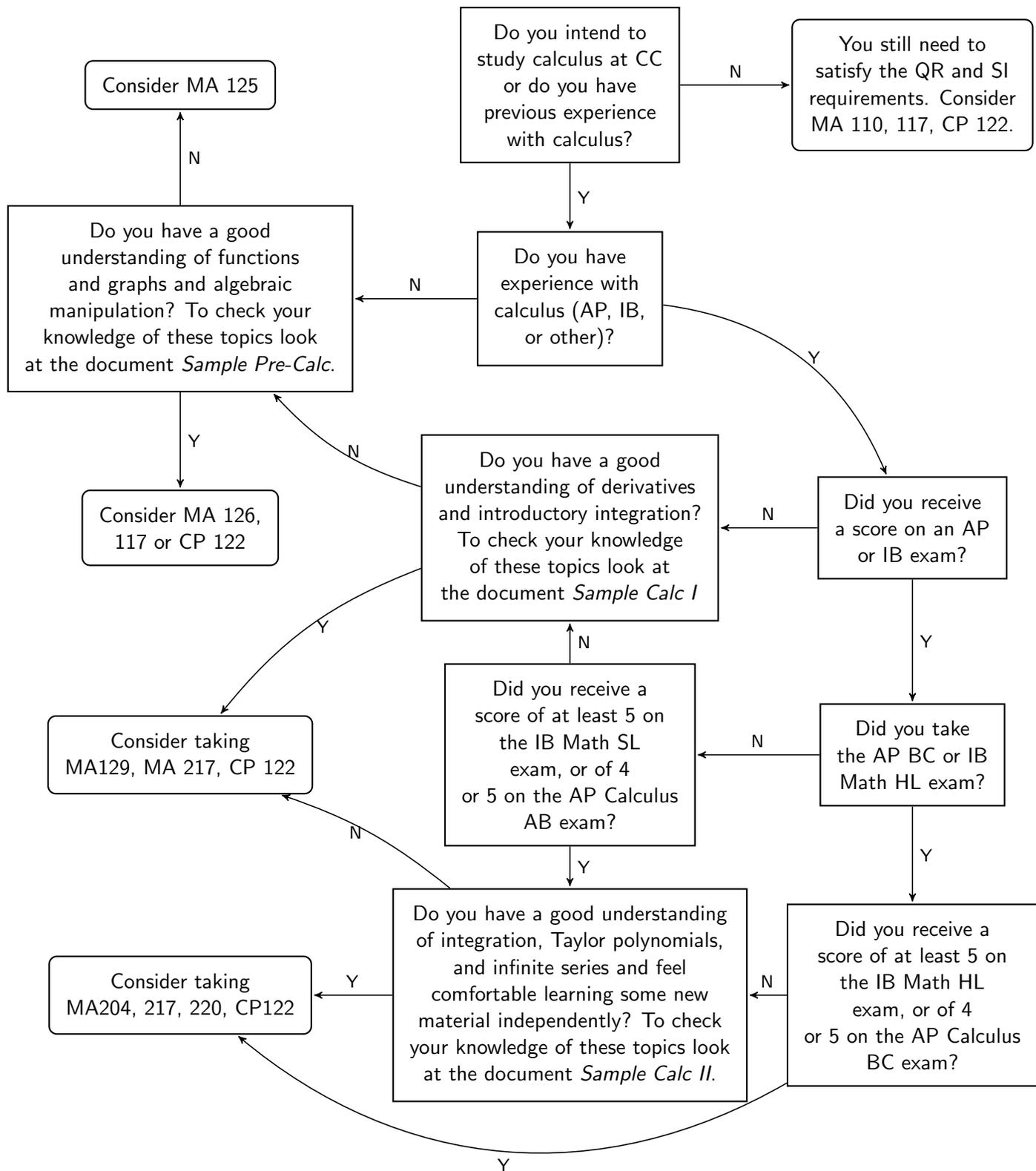
1. Make a list of majors you are considering. This is not binding, just some majors you've been thinking about.
  
2. Are any of the majors on the list below? If yes, be sure you followed yes for "do you intend to study calculus at CC?"
  
3. As you work through the decision tree if you are stuck between two possible paths, consider the following questions:
  - (a) When was the last time you took a math class?
  - (b) How independent are you in your learning? Would you feel comfortable reviewing some topics on your own?
  
4. Write down the courses the tree suggested for you. Look these up on attached document, *Choosing your First Math Class*. After reading the corresponding document, do these suggestions seem reasonable to you? If not, think about why not and talk to one of the mentors.

To help answer the question, "**do you intend to study calculus at CC?**", here is a list of majors that require Calculus 1 (MA125 or MA126) or higher (some of these majors require significantly more mathematics).

Biology, Chemistry, Computer Science, Economics, Environmental Science, Geology, International Political Economy, Mathematics, Mathematical Economics, Physics, Psychology.

The following is a list of majors that require some statistics or modeling course from the list MA117, BY220, EC200, EV228, MA256, MA217. The specific course required varies across these majors, but MA117 satisfies most of these requirements (as will MA217).

Biology, Economics, Environmental Science, Geology, International Political Economy, Mathematical Economics.

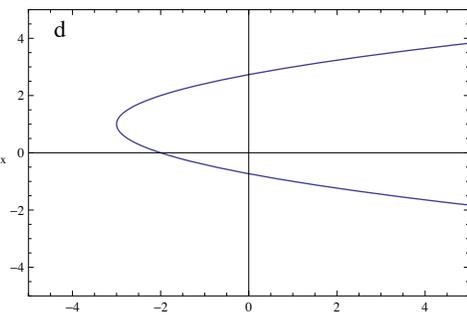
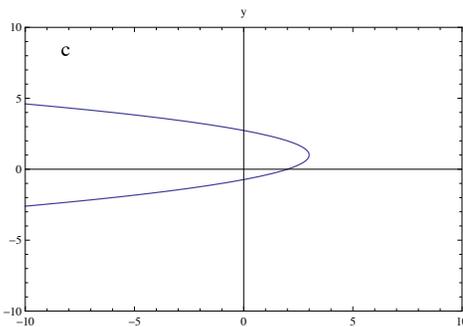
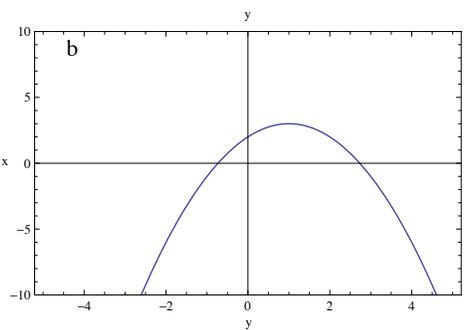
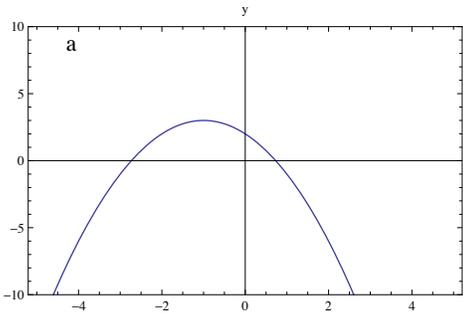


Success in quantitative disciplines at CC requires knowing your resources. Be sure to talk to your professor, the departmental paraprofessional, and the Quantitative Reasoning Center (in Tutt Library).

## Sample Pre-Calculus Questions

A student need not get 100% on the questions below in order to take Math 126 rather than Math 125. An initial score of at least 50% is good enough as long as you feel that a little review would bring your score up to at least 80%. If you feel that you would need more than a little review, then you ought to enroll in Math 125.

1. Suppose  $x^2 + 4x + 4 = 1$ . What can be concluded?
  - a.  $x = -2$
  - b.  $x = -3$
  - c.  $x = -1$
  - d.  $x = -1$  or  $x = -3$
2. Which of the following is the equation of the line passing through  $(2, -1)$  and parallel to  $y = 4x + 5$ ?
  - a.  $y = -3x + 5$
  - b.  $y = 4x - 9$
  - c.  $x + 4y = -2$
  - d.  $y + 4x = 7$
3. Which of the following is the graph of  $y = 3 - (x - 1)^2$ ?

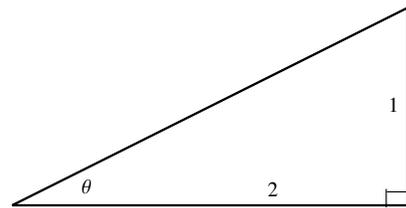


4. What is the equation of a circle of radius 2, centered at the point (3, 4)?
- $(x-3)^2 + (y-4)^2 = 4$
  - $(x+3)^2 + (y+4)^2 = 4$
  - $(x-3)^2 + (y-4)^2 = 2$
  - $x^2 + y^2 = 29$
5. If  $\frac{2}{3}$  is three less than half of  $\frac{4}{5}$  of some number, what is that number?
- $\frac{3}{5}$
  - $\frac{55}{6}$
  - $\frac{5}{3}$
  - None of the above
6. If  $f(x) = 3x^2 - 5$ , which of the following is equivalent to  $f(x+1)$ ?
- $3x^2 - 4$
  - $4x^2 - 5$
  - $3x^2 + 6x - 2$
  - $3x^2 + 3x - 2$
7. Which of the following “equations” are valid?

$$(I) \frac{x^2+1}{x} = x+1 \quad (II) \frac{a^2+b}{b} = 1 + \frac{a^2}{b} \quad (III) \frac{y^2}{y+3} = y + \frac{y^2}{3}$$

- Only I
- II and III
- Only II
- Only III

8. In the following diagram, find the value of  $\cos \theta$ :

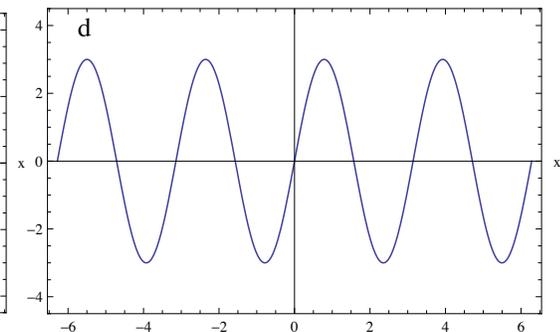
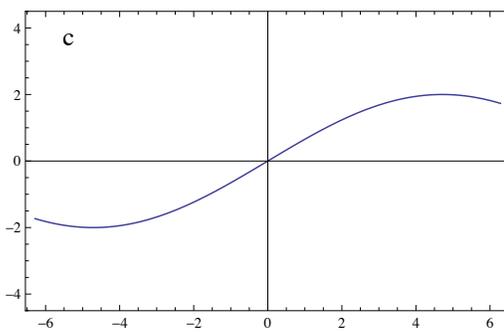
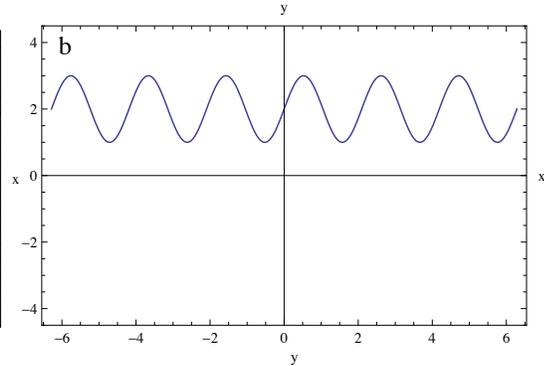
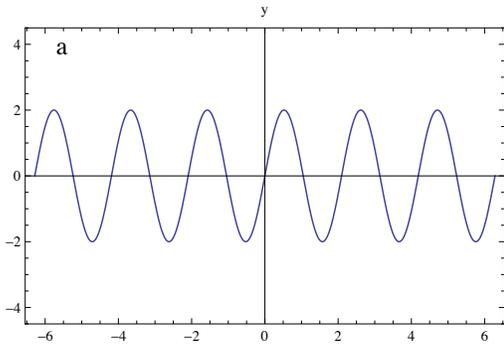


- 2
- $\frac{2}{\sqrt{5}}$
- $\frac{1}{3}$
- $\frac{2}{5}$

9. If  $\sin \theta = \frac{\sqrt{3}}{2}$ , what is  $\tan \theta$ ?

- a.  $\sqrt{3}$
- b.  $\sqrt{3}$  or  $-\sqrt{3}$
- c.  $\frac{1}{2}$  or  $-\frac{1}{2}$
- d.  $\frac{2}{\sqrt{3}}$

10. Which of the following is the graph of  $y = 2\sin(3x)$ ?



11. Solve for  $x$ :  $2^x = 100$

- a.  $\log_2 100$
- b.  $\log_{100} 2$
- c.  $\sqrt{100}$
- d.  $\log_{10} \frac{100}{2}$

12. Solve for  $x$ :  $3\log_2 x - \log_2 4 = 1$

- a. 1
- b. 2
- c.  $\frac{8}{3}$
- d.  $(\log_2 3)^{1/3}$

## Sample Calculus I Questions

A student need not get 100% on the questions below in order to skip Calculus I (Math 126) and go straight into Calculus II (Math 129). An initial score of at least 50% is good enough as long as you feel that a little review would bring your score up to at least 80%. If you feel that you would need more than a little review, then check the Pre-Calculus questions to determine if you should enroll in Math 126 or Math 125.

### 1. Derivatives Symbolically

(a) Find the derivatives of the following functions.

i.  $y = x^3 + 2x$

iii.  $y = (2x^4 + 7x)^{1/3}$

v.  $y = x \sin(x)$

ii.  $y = \cos(7x) + \frac{2}{x} + 6$

iv.  $y = e^{x^2}$

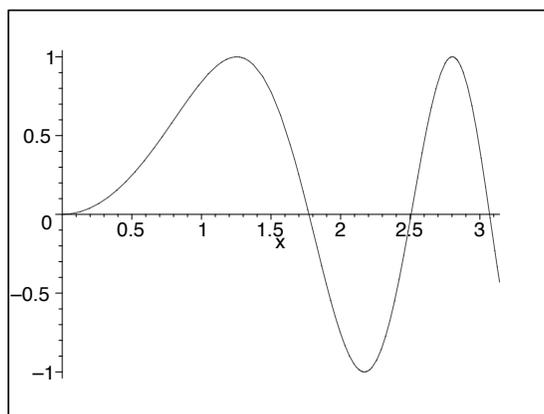
vi.  $y = \ln(x^2 + 7)$

(b) Let  $f(x) = 2x^3 - 9x^2 + 12x + 1$ .

- Find all intervals on which  $f$  is increasing.
- Find all intervals on which  $f$  is concave up.
- Find all local maxima for  $f$ .

### 2. Derivatives Graphically

Let  $f(x)$  be a function. The graph of its **derivative**,  $f'(x)$  is shown below. Give approximate answers to the questions below.



- On what interval(s) is  $f$  increasing?
- On what interval(s) is  $f$  concave up?
- At what point(s) does  $f$  reach a local maximum?
- At what point(s) does  $f'$  reach a local maximum?
- Is  $f''(1.5)$  greater than 1 or less than 1?

### 3. Derivatives Numerically

(a) Some numerical data is given for a function below.

$x$	0.0	1.0	1.7	2.3	2.5	3.5	4.2
$f(x)$	2.0	3.7	5.0	6.4	7.7	8.9	10.0

- What is the average rate of change of  $f$  on the interval  $[0, 4.2]$ ?

ii. Estimate the instantaneous rate of change of  $f$  at  $x = 3.5$  (there is more than one way to do this).

(b) The function  $g(x)$  satisfies  $g(3) = 10$  and  $g'(3) = 2$ . Estimate the value of  $g(3.1)$ .

#### 4. Derivatives Theoretically

Which of the following is a correct definition of  $f'(x)$ ?

$$(i) \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$(ii) \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$(iii) \lim_{h \rightarrow 0} \frac{f(x) + h}{h}$$

$$(iv) \lim_{x \rightarrow 0} \frac{f(x) + f(h)}{x+h}$$

#### 5. Applications

(a) Find the equation of the line tangent to  $f(x) = 3x^3 + \ln(x)$  at  $x = 1$ .

(b) A commercial nursery has 1000 yards of fencing which the owners plan to use to enclose as large a rectangular garden as possible. The garden will be bounded on one side by a barn, so no fencing is needed on that side. How large will the garden be (in square yards)?

#### 6. A Little Bit of Integration

(a) Find the following definite and indefinite integrals.

i.  $\int (x^3 + 2x) dx$

ii.  $\int_1^2 (x^3 + 2x) dx$

(b) Use a left hand sum with 4 equal subintervals to *estimate* the value of  $\int_0^2 x^2 dx$ . (Do not evaluate the integral exactly with the Fundamental Theorem of Calculus.)

## Sample Calculus II Questions

A student need not get 100% on the questions below in order to skip Calculus II (Math 129) and go straight into Linear Algebra (Math 220).<sup>\*</sup> An initial score of at least 50% is good enough as long as you feel that a little review would bring your score up to at least 80%. If you feel that you would need more than a little review, then you ought to enroll in Math 129. <sup>\*</sup>Consider Calculus 3 (Math 204) if you have no experience with partial derivatives.

### 1. Integration Techniques and the Fundamental Theorem of Calculus

(a) Use either substitution or integration-by-parts to find the following indefinite integrals.

i.  $\int x^2(5x^3 + 7)^6 dx$

iii.  $\int xe^{x^2} dx$

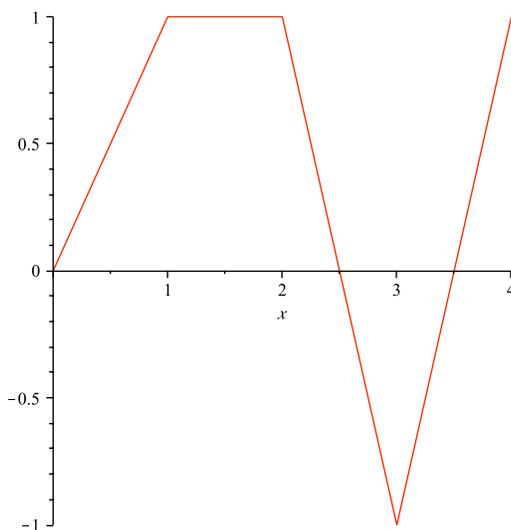
ii.  $\int x^3 \ln(x^7) dx$

iv.  $\int xe^x dx$

(b) Suppose that the function  $f(x)$  is defined by  $f(x) = \int_1^x \frac{e^t}{t} dt$ . Find  $f'(x)$ .

### 2. Anti-derivatives graphically

The graph of the function  $f(x)$  on the interval  $[0, 4]$  is shown below. Define  $F(t) = \int_0^t f(x) dx$ .



- (a) What is the value of  $F(2.5)$ ?
- (b) On what interval(s) is  $F$  increasing?
- (c) On what interval(s) is  $F$  concave up?
- (d) At what point(s) does  $F$  reach a local minimum?

### 3. Anti-derivatives numerically

The Dundas city engineers have collected some data concerning the rate  $r(t)$  of energy consumption per hour in their town. The data below represents a “typical” day in Dundas where the rate is measured in megawatts per hour,  $t$  hours after midnight.

$t$	0	4	8	12	16	20	24
$r(t)$	2.00	3.98	5.00	4.00	5.01	3.97	2.00

Make a good estimate of the total 24 hour consumption of electricity in Dundas.

#### 4. Applications of Integration

- (a) Compute the area bounded by the curves  $y = 6x$  and  $y = 3x^2$ .
- (b) Rotate the region bounded by the  $x$ -axis, the curve  $y = x^2$ , and the line  $x = 3$  about the  $x$ -axis and find the volume of the resulting solid.

#### 5. Improper Integrals

Which of the following improper integrals converge?

- (a)  $\int_1^{\infty} \frac{1}{x} dx$  (c)  $\int_0^1 \frac{1}{x^2} dx$
- (b)  $\int_1^{\infty} \frac{1}{x^2} dx$  (d)  $\int_1^{\infty} e^{-x} dx$

#### 6. Series

(a) Which of the following series converge?

- i.  $\sum_{k=0}^{\infty} 2(3^k)$  iii.  $\sum_{k=1}^{\infty} \frac{2}{k}$
- ii.  $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$  iv.  $\sum_{k=1}^{\infty} \frac{2}{k^2}$

(b) To what value do the following series converge?

- i.  $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$  ii.  $\sum_{k=2}^{\infty} 5\left(\frac{2}{3}\right)^k$

(c) Which of the following limits equals the sum of the series  $\sum_{n=0}^{\infty} a_n$ ?

- i.  $\lim_{n \rightarrow \infty} \sum_{n=0}^{\infty} a_n$  iii.  $\lim_{k \rightarrow 0} \sum_{n=k}^{\infty} a_n$
- ii.  $\lim_{k \rightarrow \infty} \sum_{n=0}^k a_n$  iv.  $\lim_{n \rightarrow \infty} a_n$

#### 7. Taylor Series

(a) The Taylor series for  $f(x) = e^x$  about  $x = 0$  is:

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

- i. What is the interval of convergence of the Taylor series for  $e^x$ ?
- ii. Use the first 4 terms of the Taylor series for  $e^x$  to estimate the value of  $e^2$  to three decimal places.

(b) The Taylor series for  $f(x) = \frac{1}{1-x}$  about  $x = 0$  is

$$1 + x + x^2 + x^3 + x^4 + \dots$$

with an interval of convergence of  $(-1, 1)$ .

- i. Find the first 4 non-zero terms of the Taylor series for  $f(x) = \frac{1}{1-2x}$ .
- ii. What is the interval of convergence for the Taylor series in the problem above?

### **Choosing Your First Math Class**

If you took an AP or IB exam or a course at another university, you should receive official notification of credit and you are encouraged to take the next appropriate class. However, our calculus sequence is not traditional and you may find the following information helpful. Regardless, here are some rough guidelines on where to start, including some options for more advanced students or those wondering what might be a good next course. These descriptions are intended to enhance the Decision tree, which is a quick and easy place to start.

#### **In calculus choose:**

**MA 125 (Precalc/Calc)** if you need Calculus 1, but your algebra and pre-calculus background is weak. In particular, choose this course if you have trouble simplifying complicated algebraic expressions involving exponents and fractions, if you have trouble seeing relationships between formulas and their graphs, or if you have not become acquainted with trig and log functions. Check the document *Sample Pre-Calculus Questions* to help you with this decision.

**MA 126 (Calc 1)** if you have a solid algebra and pre-calculus background, but have either never seen calculus, or do not feel comfortable with what you have seen.

**MA 129 (Calc 2)** if you successfully completed (and do not need help reviewing) a high school calculus course that covered techniques and applications of differentiation, and included a little introduction to integration, or if you completed calculus 2 (AP BC, for example), but did not score well on the exam, or do not feel comfortable with the work you have done.

**MA 204 (Calc 3)** if you successfully completed (and do not need help reviewing) a high school calculus 2 course (AP BC, for example), feel comfortable with that material and feel comfortable learning some new material independently.

**MA 220 (Linear Algebra)** if you successfully completed (and do not need help reviewing) a high school (or other) calculus 3 course that covered multivariable derivatives and integrals, alternate coordinate systems, and introductory vector calculus.

**MA 256 (Biological Modeling)** if you successfully completed (and do not need help reviewing) a high school (or other) calculus 3 course that covered multivariable derivatives and integrals, alternate coordinate systems, and introductory vector calculus.

We recommend against taking **MA 251** in the first year, but very strong students might consider it, especially if they take another mathematics course at CC prior to this one.

#### **In statistics choose:**

**MA 117 (Prob&Stat)** if you have not had a calculus or statistics course, and/or are not very interested in seeing the reasoning that lies behind statistical tests.

**MA 217 (Prob&Stat Modeling)** if you have successfully completed a calculus or statistics course, and like to see a little theory with your formulas. If you expect to be in a statistically needy major, you are strongly encouraged to consider this statistics course. If you have taken calculus (or higher) and feel comfortable working with mathematics at that level you are strongly encouraged to consider this statistics course.

**In Computer Science:**

**CP 122(CSI)** This is our first course in computer science and it is appropriate for students with little or no programming experience. This should be the first computer science class you take unless you have a very strong programming background in a language like Java/C/C++.

**CP 222 (CS 2)** - This course is typically the second or third computer science class taken by students. This course should be your first computer science class only if you already have a very strong programming background in a language like Java/C/C++. To take this class as a first class you should be very comfortable with basic programming techniques, conditional statements, loops, functions, objected-oriented programming, recursion, and arrays/ArrayLists.

**CP 215 (Application Design)** - This course is typically the second or third computer science class taken by students (it can be taken before or after CP222). This course should be your first computer science class only if you already have a very strong programming background in a language like Java/C/C++. To take this class as a first class you should be very comfortable with basic programming techniques, conditional statements, loops, functions, objected-oriented programming, recursion, and arrays/ArrayLists.

**Selecting a QR course:** For those who think that they will not continue in computer science, mathematics, the sciences (biology, chemistry, environmental science, geology, physics, or psychology), or economics (including mathematical economics and international political economy) should consider Math 110 or CP110 (there is at least one each year and they cover a variety of topics, Please check the course schedule to see what is offered this year). These courses typically have no prerequisites and are aimed at a general audience or any of the many other QR courses offered on campus. A listing of QR courses currently offered can be found using the online course scheduling system.